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EFFICIENCY OF POROUS COOLING

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The authors present results of a numerical analysis of the efficiency of porous cooling of a cylindrical tube with one-sided heating, accounting for the thermophysical properties being temperature dependent.

The efficiency of transpiration or porous cooling of structural elements is determined by the thermophysical and hydraulic characteristics of the porous material and by the type and mass flow rate of the coolant [1]. A well-founded choice of material and coolant ensures operational capability of thermally stressed structural elements, e.g., the arc-stabilizing porous tubes of interelectrode inserts (IEI) of plasmatrons, at the optimal coolant flow rates.

The results of investigations of processes of heat transfer and hydrodynamics in porous media have been correlated in [2, 3]. The main studies have been a filtration regimes in conditions with comparatively small temperature drop, up to 100°K through the wall thickness. The specific mass flow rate of gas through the porous wall of the IEI tube reaches 500 kg/m^2·sec (air), and the heat flux to the wall reaches 10^8 W/m^2. In these conditions the temperatures of the material and the coolant vary through the wall thickness from the temperature of the gas or liquid at the entrance to the test facility, up to the limiting working temperature of the material, e.g., from liquid nitrogen temperature (77°K) to the limiting working temperature of tungsten (2900°K) [4]. It becomes particularly important in this case to allow for the dependence of the properties of the material and the coolant on temperature in resolving the thermal and hydraulic problems of porous cooling.

The system of equations for heat transfer and hydrodynamics in a porous nondeformable medium, neglecting viscous dissipation and the kinetic energy of the gas relative to the thermal, and assuming an optically thin layer of coolant, in the one-dimensional approximation for a cylinder, has the form (the computational scheme is shown in Fig. 1):

$$\frac{1}{r}\frac{d}{dr}\lambda_{\text{eff}}\frac{dT_w}{dr}-\alpha_v(T_w-T_g)=0, \tag{1}$$

$$c_p m\frac{dT_g}{dr}=\alpha_v(T_w-T_g), \quad -\frac{dP}{dr}=\alpha\mu\frac{m}{\rho}+\beta\frac{m^2}{\rho}, \tag{2}$$

$$\rho=\frac{P}{RT_g}, \qquad m=\frac{G}{2\pi r}. \tag{3}$$

As the basic dependence of α_V on m we take

$$\alpha_v=0.029\,\mathrm{Re}^{1.84}\lambda_g/(\beta/\alpha)^2.$$

The boundary conditions are as follows:

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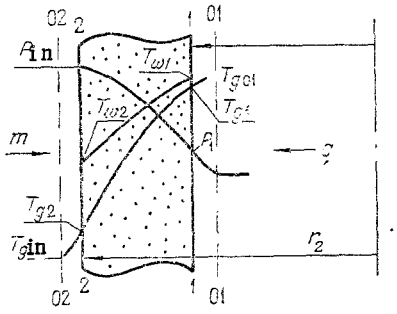

Fig. 1

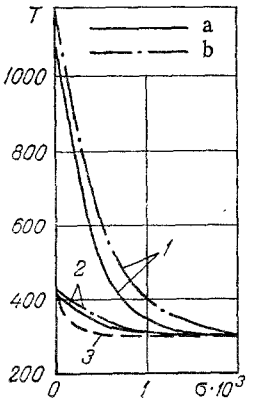

Fig. 2

Fig. 1. Computational scheme for porous cooling: sections 02 and 01 are the unperturbed region ahead of the porous wall and behind it; sections 2 and 1 are the inlet to and exit from the porous wall.

Fig. 2. Temperature distributions for the material, T_w, °K, and the gas, T_g, °K through the wall thickness δ, m ($q = 10^6$ W/m^2, $m = 0.5$ kg/m^2·sec, coolant is H_2, foam cordierite): 1) T_w; 2) T_g; 3) one-temperature model ($T_w = T_g = T$); a) λ_w, λ_g, c_p, μ—var; b) λ_w, λ_g, c_p, μ—const ($T = (T_{gin} + T_{g1})/2$).

$$r = r_1, \quad \lambda_{eff} \frac{dT_w}{dr} = q, \tag{4}$$

$$r = r_2, \quad T_{g2} = T_{gin} + \frac{\alpha_0 (T_{w2} - T_{gin})}{c_p m}, \tag{5}$$

according to [15], and

$$-\lambda_{eff} \frac{dT_w}{dr} = \alpha_0 (T_{w2} - T_{gin}), \quad P = P_{in}. \tag{6}$$

In solving a porous cooling problem it is customary to separate the thermal and hydraulic problems [1, 5]. Solutions are known for system (1)-(6), obtained by both analytical and numerical methods [1, 6], for the assumption that the thermophysical properties are constant. Solutions of the porous cooling problem have been obtained with temperature dependent properties, mainly for a one-temperature model [7] ($T_w = T_g = T$). A solution to the thermal problem was obtained in [5], allowing for temperature dependent thermophysical properties and boundary conditions of type I, but this has been checked by a practical test in only one particular example.

In contrast with this, the present paper has found a solution to the thermal-hydraulic problem, allowing for temperature dependent properties. An algorithm developed for numerical solution of the system of equations (1)-(6) by a marching method has been coded for the M-222 computer. The variation of the properties is accounted for by successive approximations. A numerical analysis of the efficiency of porous cooling is made for a tube of a

TABLE 1. Hydraulic Characteristics of the Test Materials [8]

Material	Π, %	α, m^{-2}	β, m^{-1}	$d_p \cdot 10^6$, m
W	20	$0{,}5 \cdot 10^{14}$	$0{,}42 \cdot 10^8$	5—15
Kh18N9T	42	$0{,}5 \cdot 10^{11}$	$0{,}22 \cdot 10^6$	50—150
FC	81	$0{,}115 \cdot 10^{11}$	$0{,}28 \cdot 10^6$	50—300

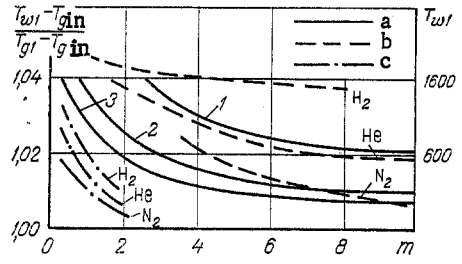

Fig. 3

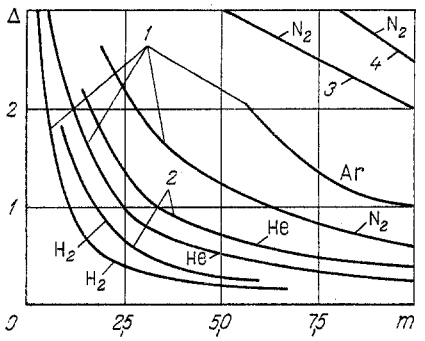

Fig. 4

Fig. 3. Dependence of the maximum material temperature T_w, °K, and excess of the material temperature over the exit gas temperature, $\eta = (T_{w1} - T_{gin})/(T_{g1} - T_{gin})$ on the nature and flow rate of coolant m, kg/m²·sec; a) $T_{w1}(N_2, q = 10^6$ W/m²); 1) FC; 2) Kh18N9T steel; 3) W; b) η(W, $q = 10^6$ W/m²); c) η(W, $q = 10^7$ W/m²).

Fig. 4. The thermal perturbation depth Δ, mm, as a function of the kind and flow rate of coolant m, kg/m²·sec; 1) FC ($q = 10^6$ W/m²); 2) FC ($q = 10^7$ W/m²); 3) Kh18N9T ($q = 10^6$ W/m²); 4) Kh18N9T ($q = 10^7$ W/m²).

plasmatron interelectrode insert (internal diameter 0.02 m, external diameter 0.024 m, height 0.05 m).

As criteria defining the efficiency of porous cooling we have chosen [1, 14]:

the maximum wall temperature T_{w1};

the temperature increase of the external wall surface above the mean exit gas temperature, $\eta = (T_{w1} - T_{gin})/(T_{g1} - T_{gin})$;

the depth of the thermal perturbation Δ (this is taken to be the porous wall thickness from the "hot" surface to the section where T_w does not exceed T_{gin} by more than 1%).

The parameter Δ is determined by the thermal conductivity of the material, by the mass flow of the coolant and by the heat flux, and, for a given limiting temperature T_{w1} it defines the optimal thickness of porous wall in the structure; for $\delta > \Delta$ the outer wall layers play practically no part in transmitting heat to the gas ($T_w \approx T_g$), and only increase the hydraulic resistance of the wall; for $\delta < \Delta$ there is an increase in the temperature T_{w1} and the heat flux from the outer surface of the IEI tube [1].

The influence of the coolant and material properties on the parameters T_{w1}, η, and Δ was analyzed for Ar, He, N_2 and H_2 on tungsten W ($\Pi = 20\%$), Kh18N9T steel ($\Pi = 42\%$), and foam cordierite (FC, $\Pi = 81\%$). The hydraulic characteristics and the thermophysical properties are given in Tables 1 and 2. The thermophysical properties of the coolant were taken from [9].

Figure 2 shows the results of analysis of the influence of temperature dependencies of the thermophysical properties of the material and the gas. For FC at a heat flux of $q = 10^6$ W/m² and cooling by hydrogen at $m = 0.5$ kg/m²·sec, the temperature difference at the

TABLE 2. Effective Thermal Conductivity of Porous Materials, λ_{eff}, W/m·K [8]

Material	T_w, K									
	300	400	500	600	700	800	900	1000	1100	1200
FC (in H_2 medium)	0,4	0,39	0,36	0,36	0,37	0,37	0,38	0,38	0,39	0,39
FC (He)	0,4	0,38	0,34	0,34	0,34	0,34	0,34	0,34	0,34	0,34
FC (Ar)	0,35	0,31	0,27	0,24	0,22	0,21	0,20	0,19	0,18	0,17
FC (N_2)	0,32	0,31	0,29	0,24	0,23	0,22	0,21	0,20	0,19	0,18
Kh18N9T	1,8	1,96	2,13	2,32	2,52	2,71	2,89	3,09	3,34	3,62
W	85	82	80	77	75	73	72	72	71	71

TABLE 3. Influence of the Inlet Heat Transfer α_0 on the Maximum Surface Temperature (W, $q = 10^7$ W/m^2, $m = 1$ kg/m^2·sec, hydrogen)

α_0 W/m^2·K	T^*_{w1}, K		
	$\delta = 10^{-3}$ m	$\delta = 3 \cdot 10^{-3}$ m	$\delta = 5 \cdot 10^{-3}$ m
10	3500	1730	1445
100	3400	1700	1440
1000	2800	1640	1420

*The temperatures for $T_W > 2000$ are around nominal (no allowance for radiation from the surface).

"hot" surface is 50°K, allowing for the temperature dependence of the properties, and not allowing for it. This value increases with increase of the level and the temperature difference through the wall. For example, with cooling of a W wall by hydrogen at $q = 10^8$ W/m^2, $m = 5$ kg/m^2·sec, the temperature difference at the surface exceeds 200°K. The distortion of the computed temperature profiles when the property dependence is allowed for increases for materials with a stronger dependence of λ on temperature, and leads to a greater reduction in the temperature head. Figure 2 also gives the results of the calculated temperature field for $\alpha_V \to \infty$, from which one obtains a limit for the one-temperature model.

The dependence of the maximum wall temperature on the thermal conductivity is shown in Fig. 3. For example, the temperatures T_{w1} of walls of W, steel and FC when cooled with nitrogen ($m = 4$ kg/m^2·sec, $q = 10^6$ W/m^2) are 500, 600, and 1200°K, respectively.

The calculated behavior of the parameter η as a function of the coolant flow rate is determined by the choice of the correlation describing heat transfer in a porous material. For the case $\alpha_V \sim m$ we note an increase in the excess temperature of the material above the gas temperature with increase in m [1]. In that reference a correlation from [11] was given, where $\alpha_V \sim m^{1.84}$. In this case an increase in m leads to a decrease in η in the range of flow rates investigated. The property temperature dependence has less influence on the behavior of η than does the choice of the heat-transfer correlation. A reduction in the computed values of the temperature head in allowing for the property dependence corresponds to a reduction in the value of η.

The increase in the wall temperature η for materials with high thermal conductivity is close to 1.0 (Fig. 3), and is appreciably increased for $\lambda_{eff} < 3$ W/m·K η, e.g., for FC $\eta \approx 3$ to 5.

The temperature field in the porous wall is appreciably affected by the effective thermal conductivity λ_{eff}. For materials with $\lambda_{eff} > 20$ W/m·K the gas is heated in a thin layer ($\sim$0.1 mm) on the inlet side of the coolant, and in the remaining region the wall temperature is close to the gas temperature. For a low thermal conductivity of the material (FC or steel Kh18N9T) the gas is heated in a thin outlet layer near the "hot" surface.

The depth of the thermal perturbation increases with increase of λ_{eff} (Fig. 4); e.g., at a flow rate of N_2 of $m = 10$ kg/m^2·sec and $q = 10^6$ W/m^2 the depth of the thermal perturbation Δ in FC is 0.6 mm, and it is 2 mm in Kh18N9T steel. An increase in Δ also results in going from gases with high heat capacity c_p and thermal conductivity λ_g (H_2 or He) to gases with smaller c_p and λ_g (N_2 and Ar). In the case of cooling of foam cordierite ($q = 10^6$ W/m^2, $m = 5$ kg/m^2·sec) by hydrogen $\Delta = 0.2$ mm, by helium $\Delta = 0.5$ mm, by nitrogen $\Delta = 1.2$ mm, and by argon $\Delta = 2.5$ mm (Fig. 4).

The literature data on heat transfer in porous media show considerable scatter [10]. Analysis of the influence of the choice of the coefficient of internal heat transfer α_V on the accuracy of calculation of the temperature fields was made by comparing results obtained using different correlations describing heat transfer in porous material [11-13]. The Re value in the chosen correlations varied from 0.65 to 1.84:

$$\mathrm{Nu} = 0{,}029\,\mathrm{Re}^{1.84} \quad [11],$$

$$\mathrm{Nu} = 0{,}005\,\mathrm{RePr} \quad [12],$$

$$\mathrm{Nu} = \left[\frac{(1-\Pi)\,\beta/\alpha}{0{,}00115}\right]^{1.33}\mathrm{Re}^{0.65} \quad [13].$$

When wall temperatures from the correlations of [11] and [12] are used in computing the fields, for $q = 10^7$ W/m^2 and $m = 1$ kg/m^2·sec (hydrogen), the results obtained are close both in temperature profile and in maximum temperature, while calculations using a correlation from [13] give a temperature field which differs markedly from those computed using correlations from [11] and [12]. The maximum wall temperature with W ($q = 10^7$ W/m^2, $m = 1$ kg/m^2·sec, $\delta = 3 \cdot 10^{-3}$ m, $\alpha_0 = 1000$ W/m^2·K) when a correlation from [13] is used is 1640°K, and it is 1160°K when correlations from [11, 12] are used. The difference in values of α_V from [11-13] is two orders of magnitude.

From analysis of the influence of the entrance heat transfer (α_0) and the wall thickness (δ) we can conclude that a flow rates of coolant corresponding to $\alpha_V > 10^7$ W/m^3·K, and for wall thicknesses larger than 10^{-3} m, a change of α_0 and δ have practically no influence on the wall temperature profile (Table 3).

A numerical analysis of the influence of α_V on the temperature field shows a need for increased accuracy and reliability of data on heat transfer in the porous interelectrode inserts of plasmatrons.

NOTATION

r, r_1, r_2, current, internal, and external radii of the porous wall, m; P, pressure, N/m^2; T_g, gas temperature, °K; T_w, wall temperature, °K; m, specific mass flow rate, kg/m^2·sec; q, specific heat flux, W/m^2; ρ, density, kg/m^3; μ, dynamic viscosity, N·sec/m^2; c_p, specific heat, J/kg·K; λ_{eff}, effective thermal conductivity of the material, W/m·K; α_V, coefficient of internal heat transfer, W/m^3·K; α_0, coefficient of initial heat transfer, W/m^2·K; R, gas constant, J/kg·K; G, mass flow rate of coolant per unit length of IEI, kg/m·sec; α, β, viscous and inertial coefficients of hydraulic resistance, m^{-2} and m^{-1}, respectively; Pr, Prandtl number; $Nu = \alpha_V(\beta/\alpha)^2/\lambda_g$, Nusselt number; $Re = m(\beta/\alpha)/\mu$, Reynolds number; Π, porosity of the material.

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A VARIATIONAL PRINCIPLE OF THE HAMILTON TYPE OF NONLINEAR PHENOMENA OF COMBINED TRANSFER

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A Lagrange function is given on the basis of which the variational principle of nonlinear combined transfer phenomena is built.

A great applied importance at the present time is possessed by mathematical models found on the use of a system of nonlinear partial differential equations of the form

$$\frac{\partial \vartheta_i}{\partial t} = \sum_{k=1}^{n} a_{ik}(\vartheta)\,\mathrm{div}\,(b_{ik}(\vartheta)\,\mathrm{grad}\,\vartheta_k) + \omega_i(\vartheta) \quad (i = 1, 2, \ldots, n). \tag{1}$$

Here $\vartheta_i = \vartheta_i(x_1, x_2, x_3, t)$ (i = 1, 2, ..., n) are functions (transfer potentials) which are continuous together with their derivatives; a_{ik}, b_{ik}, ω_i (i, k = 1, 2,..., n) are continuous functions of transfer potentials $\vartheta = (\vartheta_1, \vartheta_2, \ldots, \vartheta_n)$.

By means of this system, e.g., we describe phenomena of combined transfer of an arbitrary number of substances in a continuous medium occupying a volume v, in the presence of sources and sinks of substances and, in particular, phenomena of combined heat and mass transfer [1]. Systems of equations, being particular cases of system (1), lie at the basis of the investigation of biochemical reactions [2-4], nerve conductivity [5], development of malignant tumors [6], etc.

It is known that investigation of system (1) in each concrete case is complicated by difficulties caused by its nonlinearity, and in rare cases is completed in t closed form.

During the recent years in the solution of problems of thermodynamics of irreversible processes, a greater and greater role is played by variational methods. Allowing us partly to eliminate the difficulties connected with nonlinearity of the problems, they in a number of cases allow us to obtain effective approximate solutions of problems of combined transfer [7, 8]. It should be noted that the variational principles forming their basis usually do not use the Lagrange function, i.e., these principles do not possess a Hamiltonian form.

We consider the possibility of using a variational principle of the Hamilton type in the solution of problems leading to a system of equations (1).

Taking as $[t_0, t_1]$ an arbitrary segment of time, we write the integral of action in the form

$$I = \int_{t_0}^{t_1}\int_{v} \mathcal{L}\,dv\,dt. \tag{2}$$

We shall assume for a start that some natural boundary conditions are absent. This assumption is equivalent to the assertion that variations of transfer potentials $\delta\vartheta_i$ (i = 1, 2, ..., n) on the boundary of the volume v are zero at any time instant $t(t_0 \leq t \leq t_1)$, and also $\delta\vartheta_i|_{t_0} = \delta\vartheta_i|_{t_1} = 0$ within the volume v and on its boundary.

The system of equations (1) can be obtained by means of the variational principle $\delta I = 0$, if in (2) we take the Lagrange equation in the form

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